NAG Toolbox for MATLAB

f04af

1 Purpose

f04af calculates the accurate solution of a set of real symmetric positive-definite linear equations with multiple right-hand sides, $AX = B$, with iterative refinement, where A has been factorized by f03ae.

2 Syntax

```
[x, bb, k, ifail] = f04af(a, p, b, eps, 'n', n, 'ir', ir)
```

3 Description

To solve a set of real linear equations $AX = B$ where A is symmetric positive-definite, f04af must be preceded by a call to f03ae which computes a Cholesky factorization of A as $A = LL^{\mathrm{T}}$, where L is lower triangular. An approximation to X is then found by forward and backward substitution. The residual matrix $R = B - AX$ is then calculated using ***additional precision***, and a correction D to X is found by solving $LL^{\mathrm{T}}D = R$. X is replaced by $X + D$, and this iterative refinement of the solution is repeated until full machine accuracy has been obtained.

4 References

Wilkinson J H and Reinsch C 1971 *Handbook for Automatic Computation II, Linear Algebra* Springer–Verlag

5 Parameters

5.1 Compulsory Input Parameters

1: **a(lda,n) – double array**

lda, the first dimension of the array, must be at least **n**.

The upper triangle of the n by n positive-definite symmetric matrix A, and the subdiagonal elements of its Cholesky factor L, as returned by f03ae.

2: **p(n) – double array**

The reciprocals of the diagonal elements of L, as returned by f03ae.

3: **b(ldb,ir) – double array**

ldb, the first dimension of the array, must be at least **n**.

The n by r right-hand side matrix B.

4: **eps – double scalar**

Must be set to the value of the ***machine precision***.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The dimension of the arrays **a**, **p**. (An error is raised if these dimensions are not equal.)

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **ir – int32 scalar**

Default: The dimension of the arrays **b**, **x**, **bb**. (An error is raised if these dimensions are not equal.)

r, the number of right-hand sides.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldb, ldx, ldbb

5.4 Output Parameters

1: **x(ldx,ir) – double array**

The n by r solution matrix X.

2: **bb(ldbb,ir) – double array**

The final n by r residual matrix $R = B - AX$.

3: **k – int32 scalar**

The number of iterations needed in the refinement process.

4: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

The matrix A is too ill-conditioned to produce a correctly rounded solution.

7 Accuracy

The computed solutions should be correct to full machine accuracy. For a detailed error analysis see page 39 of Wilkinson and Reinsch 1971.

8 Further Comments

The time taken by f04af is approximately proportional to n^2r.

9 Example

```
a = [5, 7, 6, 5;
     3.130495168499706, 10, 8, 7;
     2.683281572999748, -0.8944271909999177, 10, 9;
     2.23606797749979, -6.332401570587782e-16, 2.121320343559644, 10];
p = [0.4472135954999579;
     2.236067977499794;
     0.7071067811865481;
     1.414213562373102];
b = [23;
     32;
     33;
     31];
eps = 1.111307226797642e-16;
```

```
[x, bb, k, ifail] = f04af(a, p, b, eps)
```

```
x =
     1
     1
     1
     1
bb =
     0
     0
     0
     0
k =
           3
ifail =
           0
```